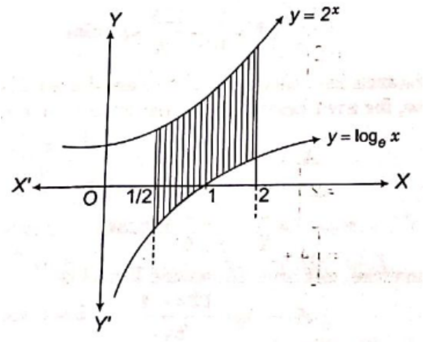
91. The required area is the shaded portion in following figure



$\therefore$ The required area $= \int_{1/2}^{2} (2^x - \log x)\, dx = \left(\frac{2^x}{\log 2} - (x \log x - x) \right)_{1/2}^{2}$

$= \left(\frac{4-\sqrt{2}}{\log 2} - \frac{5}{2}\log 2 + \frac{3}{2} \right)$ square units.

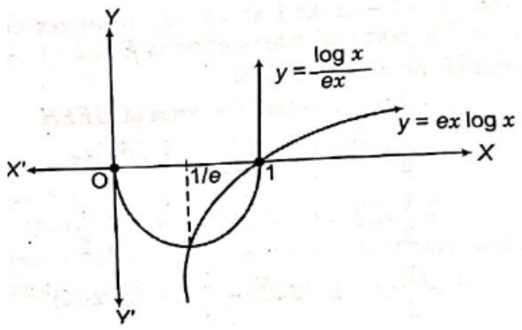
92. Both the curves are defined for $x > 0$.
Both are positive when $x > 1$and negative when $0 < x < 1$.
We know that, $\lim\limits_{x \to 0^+} (\log x) \to -\infty$

Hence, $\lim\limits_{x \to 0^+} \frac{\log x}{ex} \to -\infty$. Thus, Y-axis is asymptote of second curve.

And $\lim\limits_{x \to 0^+} ex \log x$ $[(0) \times \infty \text{ form}]$

$= \lim\limits_{x \to 0^+} \frac{e \log x}{1/x}$ $\left[-\frac{\infty}{\infty} \text{form} \right]$

$= \lim\limits_{x \to 0^+} \frac{e\left(\frac{1}{x}\right)}{\left(-\frac{1}{x^2}\right)} = 0$ [Using L'Hospital's rule]



Thus, the first curve starts from (0, 0) but does not include (0, 0).

Now, the given curves intersect, therefore $ex \log x = \frac{\log x}{ex}$

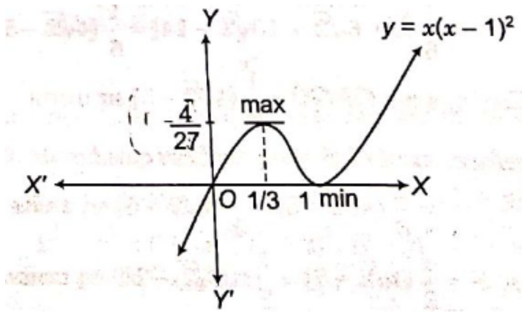
i.e. $\left(e^2 x^2 - 1\right) \log x = 0$

$\Rightarrow$ $x = 1, \frac{1}{e}$ $[\because \ x > 0]$

$\therefore$ The required area

$= \int_{1/e}^{1} \left(\frac{(\log x)}{ex} - ex \log x \right) d = \frac{1}{e}\left[\frac{(\log x)^2}{2} \right]_{1/e}^{1} - e\left[\frac{x^2}{4}(2\log x - 1) \right]_{1/e}^{1} = \left(\frac{e^2 - 5}{4e} \right)$ square units

93. $y_{max} = 2$; $y_{min} = 0$; Area $= \frac{10}{3}$

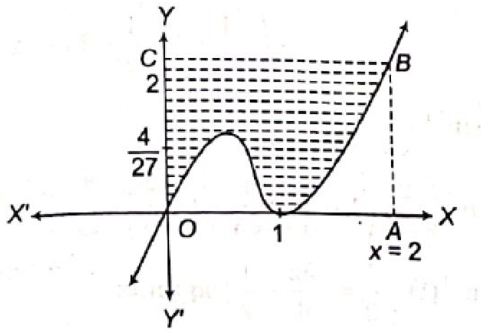


Given, $y = x(x-1)^2 \Rightarrow \frac{dy}{dx} = x \cdot 2(x-1) + (x-1)^2$

$= (x-1)\cdot(2x + x - 1) = (x-1)(3x-1) = 0 \quad \Rightarrow x = 1, \frac{1}{3}$

x	y
0	0
$\frac{1}{3}$	$\frac{4}{27}$
1	0
2	2
$y_{max} = 2$	$y_{min} = 0$

Now, to find the area bounded by the curve $y = x(x-1)^2$, the Y-axis and line $x = 2$.



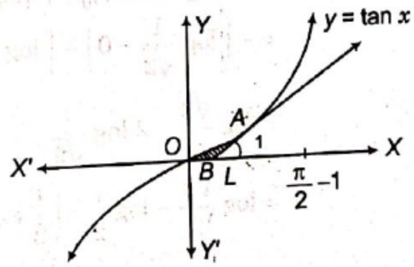
$\therefore$ Required area = Area of square OABC $-\int_0^2 y\,dx = 2 \times 2 - \int_0^2 x(x-1)^2\,dx$

$$= 4 - \left[\left[\frac{x(x-1)^3}{3}\right] - \frac{1}{3}\int_0^2 (x-1)^3 \cdot 1dx\right]$$

$$= 4 - \left[\frac{x}{3}(x-1)^3 - \frac{(x-1)^4}{12}\right]_0^2 = 4 - \left[\frac{2}{3} - \frac{1}{12} + \frac{1}{12}\right] = \frac{10}{3} \text{ square units.}$$

94. Given, $y = \tan x \quad \Rightarrow \quad \frac{dy}{dx} = \sec^2 x \quad \therefore \quad \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 2.$

Hence, equation of tangent at $A\left(\frac{\pi}{4}, 1\right)$ is $\frac{y-1}{x-\pi/4} = 2 \quad \Rightarrow \quad y - 1 = 2x - \frac{\pi}{2}$

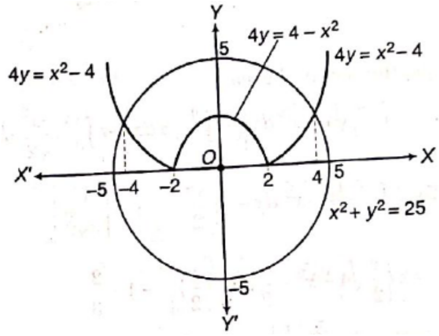


$\Rightarrow \quad (2x - y) = \left(\frac{\pi}{2} - 1\right)$

Required area is OABO $= \int_0^{\pi/4} (\tan x)\, dx$ − area of ΔALB

$= [\log|\sec x|]_0^{\pi/4} - \frac{1}{2} \cdot BL \cdot AL = \log\sqrt{2} - \frac{1}{2}\left(\frac{\pi}{4} - \frac{\pi - 2}{4}\right) \cdot 1 = \left(\log\sqrt{2} - \frac{1}{4}\right)$ square units.

95. Given curves, $x^2 + y^2 = 25, 4y = |4 - x^2|$ could be sketched as below, whose points of intersection are $x^2 + \frac{(4 - x^2)^2}{16} = 25$



$\Rightarrow \quad (x^2 + 24)(x^2 - 16) = 0 \quad \Rightarrow \quad x = \pm 4$

$\therefore$ Required area $= 2\left[\int_0^4 \sqrt{25 - x^2}\, dx - \int_0^2 \left(\frac{4 - x^2}{4}\right) dx - \int_2^4 \left(\frac{x^2 - 4}{4}\right) dx\right]$

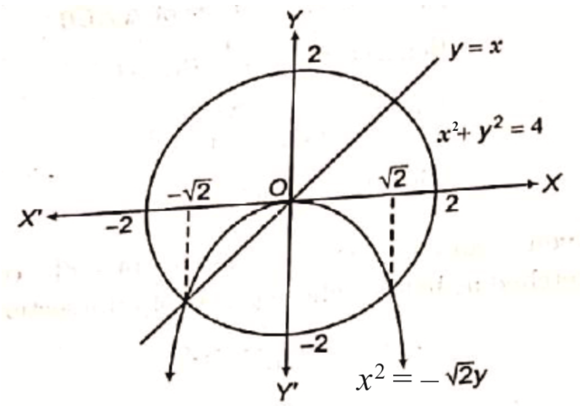
$$= 2\left[\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^4 - \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3} - 4x\right]_2^4\right]$$

$$= 2\left[\left[6 + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)\right] - \frac{1}{4}\left[\frac{x^3}{3} - 4x\right]_2^4\right]$$

$$= 2\left[\left[6 + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right)\right] - \frac{1}{4}\left[8 - \frac{8}{3}\right] - \frac{1}{4}\left[\left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right)\right]\right]$$

$$= 2\left[6 + \frac{25}{2}\sin^{-1}\left(\frac{4}{5}\right) - \frac{4}{3} - \frac{4}{3} - \frac{4}{3}\right] = \left[4 + 25\sin^{-1}\left(\frac{4}{5}\right)\right] \text{ square units}$$

96. Given curves are $x^2 + y^2 = 4, x^2 = -\sqrt{2}y$ and $x = y$.



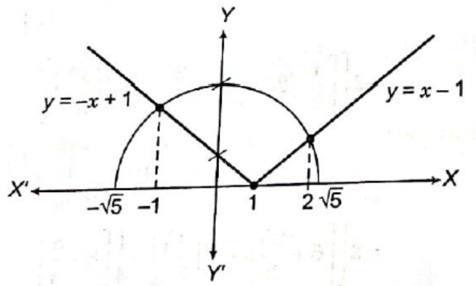
Thus, the required area $=\left|\int_{-\sqrt{2}}^{\sqrt{2}}\sqrt{4-x^2}\,dx\right|-\left|\int_{-\sqrt{2}}^{0}x\,dx\right|-\left|\int_0^{\sqrt{2}}\frac{-x^2}{\sqrt{2}}\,dx\right|$

$=2\int_0^{\sqrt{2}}\sqrt{4-x^2}\,dx-\left|\left(\frac{x^2}{2}\right)_{-\sqrt{2}}^{0}\right|-\left|\frac{x^3}{3\sqrt{2}}\right|_0^{\sqrt{2}}$

$=2\left\{\frac{x}{2}\sqrt{4-x^2}-\frac{4}{2}\sin^{-1}\frac{x}{2}\right\}_0^{\sqrt{2}}-1-\frac{2}{3}$

$=(2-\pi)-\frac{5}{3}=\left(\frac{1}{3}-\pi\right)$ square units.

97. Given curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ could be sketched as shown, whose point of intersection are



$5-x^2=(x-1)^2$

$5-x^2=x^2-2x+1$

$\Rightarrow\quad 2x^2-2x-4=0$

$x=2,-1$

$\therefore\quad$ Required area $=\int_{-1}^{2}\sqrt{5-x^2}dx-\int_{-1}^{1}(-x+1)\,dx-\int_1^2(x-1)dx$

$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_{-1}^{2}-\left[\frac{-x^2}{2}+x\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_1^2$

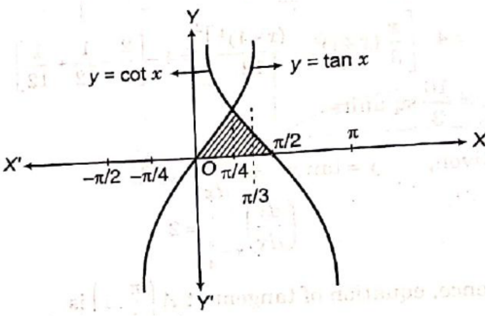
$=\left(1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right)-\left[-1+\frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]-\left(-\frac{1}{2}+1+\frac{1}{2}+1\right)-\left(2-2-\frac{1}{2}+1\right)$

$=\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}}\right)-\frac{1}{2}=\frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{4}{5}}\right)-\frac{1}{2}$

$=\frac{5}{2}sin^{-1}(1)-\frac{1}{2}=\left(\frac{5\pi}{4}-\frac{1}{2}\right)$ square units.

98. Given, $y=\begin{cases}\tan x, -\frac{\pi}{3}\le x\le\frac{\pi}{3}\\ \cot x, \frac{\pi}{6}\le x\le\frac{\pi}{2}\end{cases}$

Which could be plotted as Y-axis.



$\therefore$ Required area $= \int_0^{\pi/4} (\tan x)\, dx + \int_{\pi/4}^{\pi/3} (\cot x)\, dx = [-\log|\cos x|]_0^{\pi/4} + [\log \sin x]_{\pi/4}^{\pi/3}$

$= -\left(\log \frac{1}{\sqrt{2}} - 0\right) + \left(\log \frac{\sqrt{3}}{2} - \log \frac{1}{\sqrt{2}}\right) = \log \frac{\sqrt{3}}{2} - 2\log \frac{1}{\sqrt{2}}$

$= \log \frac{\sqrt{3}}{2} - \log \frac{1}{2} = \left(\frac{1}{2}\log_e 3\right)$ square units.

99. Area = 4, $a = 2\sqrt{2}$

Here, $\int_2^a \left(1 + \frac{8}{x^2}\right) dx = \int_a^4 \left(1 + \frac{8}{x^2}\right) dx$

$\Rightarrow \quad \left[x - \frac{8}{x}\right]_2^a = \left[x - \frac{8}{x}\right]_a^4 \quad \Rightarrow \quad a - \frac{8}{a} + 2 = 2 - a + \frac{8}{a} \quad \Rightarrow \quad 2a - \frac{16}{a} = 0$

$\Rightarrow \quad 2(a^2 - 8) = 0 \quad \Rightarrow \quad a = \pm 2\sqrt{2}$ [neglecting – sign]

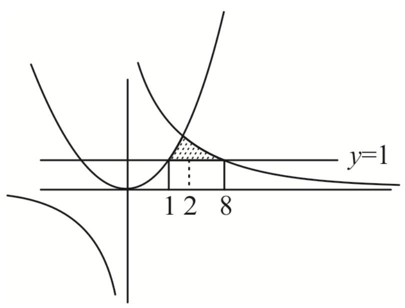
$\therefore \quad a = 2\sqrt{2}$

100.(9/8) The point of intersection of the curves $x^2 = 4y$ and $x = 4y - 2$ could be sketched are $x = -1$ and $x = 2$.

$\therefore$ Required area $= \int_{-1}^{2} \left\{\left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right)\right\} dx = \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left[\left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right] = \frac{1}{4}\left[\frac{10}{3} - \left(\frac{-7}{6}\right)\right] = \frac{1}{4} \cdot \frac{9}{2} = \frac{9}{8}$ square units.

101.(D)



$\int_1^2 x^2 dx + \int_2^8 \frac{8}{x} dx - 7 = \frac{7}{3} + 8\ell n 4 - 7 = \left(16\ell n 2 - \frac{14}{3}\right)$ sq. units

102.(4.00) $2I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{1+e^{\sin x}}{(1+e^{\sin x})(2-\cos 2x)}dx$

$I = \frac{2}{\pi}\int_0^{\pi/4} \frac{dx}{2-\cos^2 x} = \frac{2}{\pi}\int_0^{\pi/4} \frac{dx}{1+2\sin^2 x} = \frac{2}{\pi}\int_0^{\pi/4} \frac{\sec^2 x\ dx}{1+3\tan^2 x} = \frac{2}{\pi}\int_0^1 \frac{dt}{1+3t^2} = \frac{2\sqrt{3}}{3\pi}\left|\tan^{-1}\frac{t}{1/\sqrt{3}}\right|_0^1$

$= \frac{2\sqrt{3}}{3\pi}\cdot\frac{\pi}{3}$

$I = \frac{2\sqrt{3}}{9}$; $27I^2 = 27\frac{4.3}{81} = 4$

103.(0.50) $I = \int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5}d\theta$ $\Rightarrow$ $I = 3\int_0^{\pi/2} \frac{\sqrt{\sin\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$

$\Rightarrow$ $2I = 3\int_0^{\pi/2} \frac{1}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^4}d\theta$ $\Rightarrow$ $2I = 3\int_0^{\pi/2} \frac{\sec^2\theta}{\left(1+\sqrt{\tan\theta}\right)^4}d\theta$

Let $\tan\theta = t^2$

$\Rightarrow$ $2I = 3\int_0^{\infty} \frac{2t\ dt}{(1+t)^4}$ $\Rightarrow$ $I = \frac{1}{2}$